

The graph of $f(t)$ is shown on the right. **NOTE: The graph consists of a quarter circle, a semi-circle, a line segment, another quarter circle, and another line segment.**

SCORE: ____ / 20 PTS

Let $g(x) = \int_5^x f(t) dt$.

[a] Find $g(-2)$.

$$\int_5^{-2} f(t) dt = - \int_{-2}^5 f(t) dt = - \left[\frac{1}{4}\pi(2)^2 - \frac{1}{2}(3)(3) - \frac{1}{4}\pi(2)^2 - 2 \right] = \frac{13}{2}$$

[b] Find $g'(-4)$.

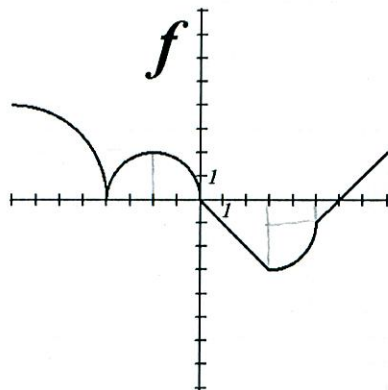
$$\frac{1}{2} f(-4) = 0$$

[c] Find $g''(2)$.

$$2 f'(2) = -1$$

[d] Find the x -coordinates of all local maxima of g .

$g'(x) = f(x)$ CHANGES FROM POSITIVE TO NEGATIVE ③
 ② $x = 0$ ③



Find $\frac{d}{dx}(\tanh^{-1} x^2 + \operatorname{sech} 3x)$.

SCORE: ____ / 15 PTS

You may use any hyperbolic identities or the derivatives of any hyperbolic functions without proving them.

$$\begin{aligned} & \frac{1}{1-(x^2)^2} \cdot 2x + (-\operatorname{sech} 3x \tanh 3x)(3) \\ &= \frac{2x}{1-x^4} - 3\operatorname{sech} 3x \tanh 3x \end{aligned}$$

Find $\lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \frac{1}{3 + \frac{4i}{n}}$ by finding the corresponding definite integral, and evaluating that integral.

SCORE: ____ / 15 PTS

$$\begin{aligned} \int_3^7 \frac{1}{x} dx &= \ln|x| \Big|_3^7 \\ &= \ln 7 - \ln 3 \\ &= \ln \frac{7}{3} \end{aligned}$$

$$a + i\Delta x = 3 + \frac{4i}{n}$$

$$a = 3, \Delta x = \frac{4}{n} = \frac{b-a}{n}$$

$$\frac{4}{n} = \frac{b-3}{n}$$

$$b = 7$$

$$f(a + i\Delta x) = f\left(3 + \frac{4i}{n}\right) = \frac{1}{3 + \frac{4i}{n}}$$

$$f(x) = \frac{1}{x}$$

The velocity of a car at time t hours after 1 pm is given by $v(t) = t^2 - 3t - 4$ miles per hour.

SCORE: ____ / 15 PTS

Find the total distance travelled by, **NOT THE DISPLACEMENT OF**, the car from 2 pm to 5 pm.

$$\int_1^4 |v(t)| dt = \int_1^4 |t^2 - 3t - 4| dt$$

$$t=1 \quad t=4$$

$$\stackrel{\textcircled{1}}{=} \int_1^4 \stackrel{\textcircled{5}}{(-t^2 + 3t + 4)} dt$$

$$\stackrel{\textcircled{1}}{=} \left(-\frac{1}{3}t^3 + \frac{3}{2}t^2 + 4t \right) \Big|_1^4$$

$$= -\frac{1}{3}(64-1) + \frac{3}{2}(16-1) + 4(4-1)$$

$$= -21 + \frac{45}{2} + 12 = \frac{27}{2} \text{ miles} \stackrel{\textcircled{1}}{}$$

$\stackrel{\textcircled{3}}{\underbrace{\hspace{1cm}}}$ $\stackrel{\textcircled{3}}{\underbrace{\hspace{1cm}}}$

$$t^2 - 3t - 4 \\ = (t-4)(t+1)$$

$$v(t) > 0 \quad < 0 \quad > 0 \\ \hline \quad \quad -1 \quad \quad \quad 4$$

Prove that $\frac{\pi}{2} \leq \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \leq \frac{2\pi}{3}$.

SCORE: ____ / 15 PTS

ON $[1, \sqrt{3}]$, (5) $\frac{\pi}{4} \leq \tan^{-1} x \leq \frac{\pi}{3}$

$$\frac{\pi}{4} x^3 \leq x^3 \tan^{-1} x \leq \frac{\pi}{3} x^3$$

$$\begin{aligned} & \int_1^{\sqrt{3}} x^3 \, dx \\ &= \left. \frac{1}{4} x^4 \right|_1^{\sqrt{3}} \\ &= \frac{1}{4} (9 - 1) = 2 \end{aligned}$$

(4) $\int_1^{\sqrt{3}} \frac{\pi}{4} x^3 \, dx \leq \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \leq \int_1^{\sqrt{3}} \frac{\pi}{3} x^3 \, dx$

$$\frac{\pi}{2} = 2\left(\frac{\pi}{4}\right) \leq \int_1^{\sqrt{3}} x^3 \tan^{-1} x \, dx \leq 2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$$

(4)

Evaluate the following integrals. If a definite integral cannot be evaluated using the Fundamental Theorem of Calculus, write "FTC DOES NOT APPLY", and explain why not with proper justification.

SCORE: ____ / 55 PTS

[a] $\int_{-1}^2 \frac{1-x}{\sqrt{5+6x}} dx$

③ FTC DOES NOT APPLY —

③ DISCONTINUOUS (DOES NOT EXIST) @ $x = -\frac{5}{6}$

④

[b] $\int_{-3}^3 \frac{7t^2}{(4-t^2)^5} dt$

③ FTC DOES NOT APPLY —

③ DISCONTINUOUS @ $t = \pm 2$

④

[c] $\int \frac{5e^{2y}}{\sqrt{1-e^{4y}}} dy$

④ $u = e^{2y}$
 $du = 2e^{2y} dy$
 $\frac{5}{2} du = 5e^{2y} dy$

⑤ $\int \frac{\frac{5}{2} du}{\sqrt{1-u^2}}$

⑤ $= \frac{5}{2} \sin^{-1} u + C$

$= \frac{5}{2} \sin^{-1} e^{2y} + C$ ②

④

[d] $\int_{-\pi}^{\pi} x^5 \cos x dx$

④ $(-x)^5 \cos(-x) = -x^5 \cos x$

④ ODD, CONTINUOUS ③

INTEGRAL = 0

④